MMAN2300 2013-10-05 Assignment C

Given: m_P=440 g, R=42 mm, H=38 mm, L=147 mm, m_R=470 g, $\bar{I}_R$=1.75 gm^2.

$$\omega_{AB}=3500 \text{ rpm}=3500\times\frac{2\pi}{60}=366.52 \text{ rad/s}.$$

$$\frac{L}{R}=3.5,\quad \left(\frac{L}{R}\right)^2=12.25,\quad \frac{R}{L}=0.2857,\quad \left(\frac{R}{L}\right)^2=0.08163,\quad L-H=0.109,\quad \frac{H}{L}=0.2585.$$

$$\omega_{BC}=\frac{-\omega_{AB}\sin\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}}=\frac{-366.52\sin\theta}{\sqrt{12.25-\cos^2\theta}},\quad \omega_{BC}^2=\frac{1.343\times10^5\sin\theta}{12.25-\cos^2\theta}.$$

$$\sin\phi=\frac{R}{L}\cos\theta=0.2857\cos\theta,\quad \cos\phi=\sqrt{1-\left(\frac{R}{L}\right)^2\cos^2\theta}=\sqrt{1-0.08163\cos^2\theta}.$$

$$\alpha_{BC}=\frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}}\cdot\omega_{AB}^2=\frac{-1.511\times10^6\cos\theta}{[12.25-\cos^2\theta]^{\frac{3}{2}}}.$$

$$a_{C_y}=-R\omega_{AB}^2\left\{\frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos^2\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}}+\frac{\sin^2\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}}+\sin\theta\right\}$$

$$=-5642\left\{\frac{-11.25\cos^2\theta}{[12.25-\cos^2\theta]^{\frac{3}{2}}}+\frac{\sin^2\theta}{\sqrt{12.25-\cos^2\theta}}+\sin\theta\right\}$$

f) Find an expression for the kinetic energy of the connecting rod as a function of crank angle.

$$T = \frac{1}{2} m_R \bar{v}^2 + \frac{1}{2} I_C \omega_{BC}^2.$$

$$\bar{\mathbf{v}} = v_C + \omega_{\mathbf{BC}} \times r_{D/C}, \qquad I_C = \bar{I}_R + m_R\, r_{D/C}^2 = 1.75 \times 10^{-3} + 0.470 \times 0.109^2 = 7.334 \times 10^{-3}.$$

$$v_{C/B} = \omega_{\mathbf{BC}} \times r_{CB} = \mathbf{k}\, \omega_{BC} \times L\,(-\mathbf{i}\sin\phi + \mathbf{j}\cos\phi) = \omega_{BC} L\,(-\mathbf{j}\sin\phi - \mathbf{i}\cos\phi)$$

$$= \omega_{BC} L \left(-\mathbf{j}\frac{R}{L}\sin\theta - \mathbf{i}\sqrt{1 - \left(\frac{R}{L}\right)^2 \cos^2\theta} \right) \qquad = -\omega_{BC}\left(\mathbf{j}R\cos\theta + \mathbf{i}\sqrt{L^2 - R^2\cos^2\theta}\right).$$

$$v_B = v_A + \omega_{\mathbf{AB}} \times r_{B/A} = 0 + \mathbf{k}\,\omega_{AB} \times R(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) = \omega_{AB} R(\mathbf{j}\cos\theta - \mathbf{i}\sin\theta).$$

$$v_C = v_B + v_{C/B} = \omega_{AB} R(\mathbf{j}\cos\theta - \mathbf{i}\sin\theta) - \omega_{BC}\left(\mathbf{j}R\cos\theta + \mathbf{i}\sqrt{L^2 - R^2\cos^2\theta}\right).$$

$$\because v_{C_x} = 0, \quad \therefore v_C = \mathbf{j}\ v_{C_y} = \mathbf{j}(\omega_{AB} - \omega_{BC}) R\cos\theta.$$

$$r_{D/C} = (L - H)(\mathbf{i}\sin\phi - \mathbf{j}\cos\phi).$$

$$\bar{\mathbf{v}} = v_C + \omega_{\mathbf{BC}} \times r_{D/C} = \mathbf{j}(\omega_{AB} - \omega_{BC}) R\cos\theta + \mathbf{k}\,\omega_{BC} \times (L - H)(\mathbf{i}\sin\phi - \mathbf{j}\cos\phi)$$

$$= \mathbf{j}(\omega_{AB} - \omega_{BC}) R\cos\theta + \omega_{BC}(L - H)(\mathbf{j}\sin\phi + \mathbf{i}\cos\phi).$$

$$\bar{v}^2 = [\omega_{BC}(L - H)\cos\phi]^2 + [(\omega_{AB} - \omega_{BC}) R\cos\theta + \omega_{BC}(L - H)\sin\phi]^2$$

$$= \left[\frac{-366.52\sin\theta}{\sqrt{12.25 - \cos^2\theta}} \cdot 0.109 \cdot \sqrt{1 - 0.08163\cos^2\theta} \right]^2$$

$$+ \left[\left(366.52 + \frac{366.52\sin\theta}{\sqrt{12.25 - \cos^2\theta}}\right) \cdot 0.042\cos\theta - \frac{366.52\sin\theta}{\sqrt{12.25 - \cos^2\theta}} \cdot 0.109 \cdot 0.2857\sin\theta \right]^2$$

$$= \left[\frac{-39.95\sin\theta}{\sqrt{12.25 - \cos^2\theta}} \sqrt{1 - 0.08163\cos^2\theta} \right]^2 + \left[15.39\left(1 + \frac{\sin\theta}{\sqrt{12.25 - \cos^2\theta}}\right)\cos\theta - \frac{11.47\sin^2\theta}{\sqrt{12.25 - \cos^2\theta}} \right]^2$$

$$= \frac{1596\sin^2\theta}{12.25 - \cos^2\theta}(1 - 0.08163\cos^2\theta) + \left[15.39\cos\theta + \frac{15.39\sin\theta\cos\theta}{\sqrt{12.25 - \cos^2\theta}} - \frac{11.47\sin^2\theta}{\sqrt{12.25 - \cos^2\theta}} \right]^2$$

$$= \frac{1596\sin^2\theta}{12.25 - \cos^2\theta}(1 - 0.08163\cos^2\theta) + \left[15.39\cos\theta + \frac{\sin\theta}{\sqrt{12.25 - \cos^2\theta}}(15.39\cos\theta - 11.47\sin\theta) \right]^2$$

Putting all together…

$$T = \frac{1}{2} m_R \bar{v}^2 + \frac{1}{2} I_C \omega_{BC}^2 = 0.53\ \bar{v}^2 + 7.334 \times 10^{-3}\ \omega_{BC}^2.$$

$$= 0.53 \left\{ \frac{1596\sin^2\theta}{12.25 - \cos^2\theta}(1 - 0.08163\cos^2\theta) + \left[15.39\cos\theta + \frac{\sin\theta}{\sqrt{12.25 - \cos^2\theta}}(15.39\cos\theta - 11.47\sin\theta) \right]^2 \right\}$$

$$+ 7.334 \times 10^{-3} \left(\frac{-366.52\sin\theta}{\sqrt{12.25 - \cos^2\theta}} \right)^2.$$

$$= \frac{845.9\sin^2\theta}{12.25 - \cos^2\theta}(1 - 0.08163\cos^2\theta) + \left[8.157\cos\theta + \frac{\sin\theta}{\sqrt{12.25 - \cos^2\theta}}(8.157\cos\theta - 6.079\sin\theta) \right]^2$$

$$+ \frac{985.2\sin^2\theta}{12.25 - \cos^2\theta}.$$

$$= \frac{1831\sin^2\theta - 69.05\sin^2\theta\cos^2\theta}{12.25 - \cos^2\theta} + \left[8.157\cos\theta + \frac{\sin\theta}{\sqrt{12.25 - \cos^2\theta}}(8.157\cos\theta - 6.079\sin\theta) \right]^2.$$